An Algorithmic Framework for Wideband Time-Frequency Processing

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Discussion Outline

A. Generalized detection/estimation proble: Pre-D processing is critical

B. Gabor versus Wavelet filter decompositions

C. A universal design for Gabor filter bank processing

D. Analysis/synthesis windows: Modern approaches

E. Time invariant filtering as a familiar special case

F. Symmetry relations, Hilbert Transform, analytic signals

G. Bandpass concepts : instantaneous frequency and magnitude

H. Joint time-frequency signals, processing gain, cross correlation

I. Example algorithms: set-on receivers, cyclostationary analysis

J. Conclusions, feasibility, and hardware for Gabor processing

Optimum Signal Processor

Biorthogonal Preprocessor

- Transforms to more compact spaces
- Eigenfunctions and principal component
- DFT for efficient convolutions & filters
- Wavelets for scale invariant problems

$[S(z)]_{N<<M}$

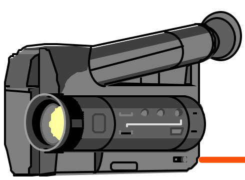
Statistical Decision Processor

- Classical rules (Bayes, Neyman)
- Multivariate Systems (Fuzzy, Neural)
- Adaptive algorithms (Genetic, LMS)

$[A_k]$

Host CPU

- Final Review: MMI
- Archive & Export



$[s(t)]_M$

References
Van Trees
DiFranco & Rubin

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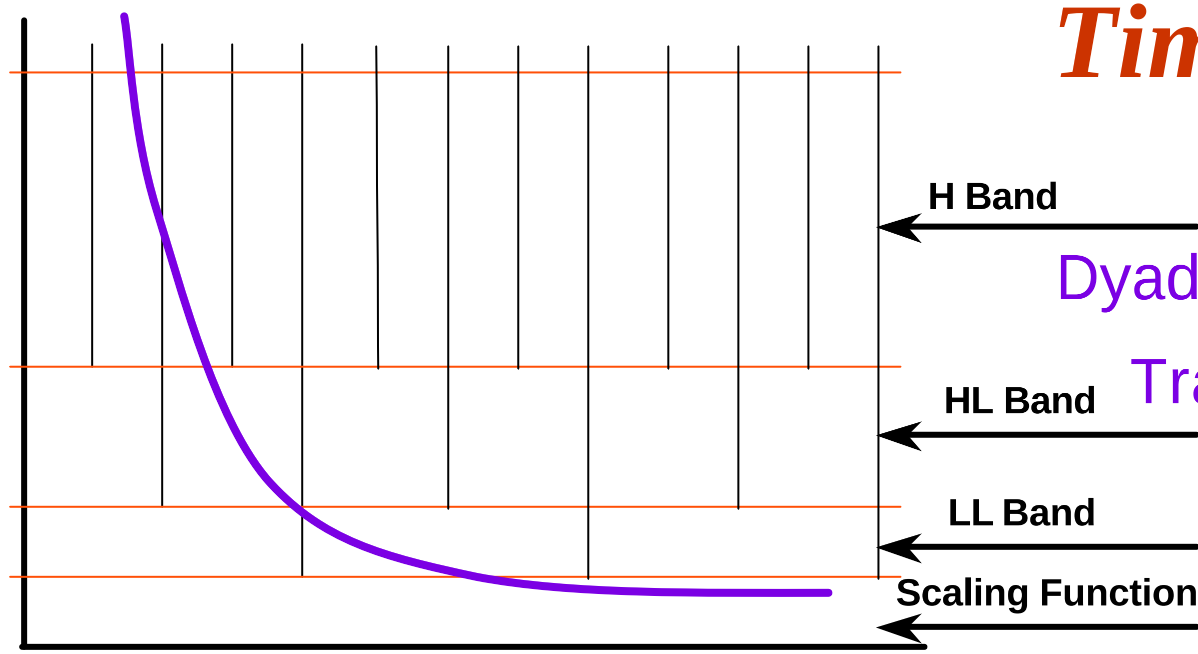
Key Detection & Estimation Points

- **A general detection & estimation process is composed of two parts:**
  - Signal preprocessor, using linear transformation with either an orthogonal (Fourier, Wavelet) or biorthogonal (e.g. - Gabor, Wavelet) expansion
  - Statistical decision processor, which classically was either Bayesian or Neyman-Pearson, but now has been supplemented by Neural, Fuzzy, and Adaptive techniques
- **The preprocessor is critical to good performance:**
  - Reduces the signal space, concentrating energy in a few of the transform coefficients
  - Allows separation of desired signals from unwanted noise and interference
- **Once the signal(s) have been separated and concentrated, the nature of the decision processor will be determined by the number of remaining dimensions, available SNR, and time variations in the statistics.**
  - Starting with a neural approach, CAPRIS moved to a simple binary threshold once the preprocessor included both prewhitening & Hough concentration.

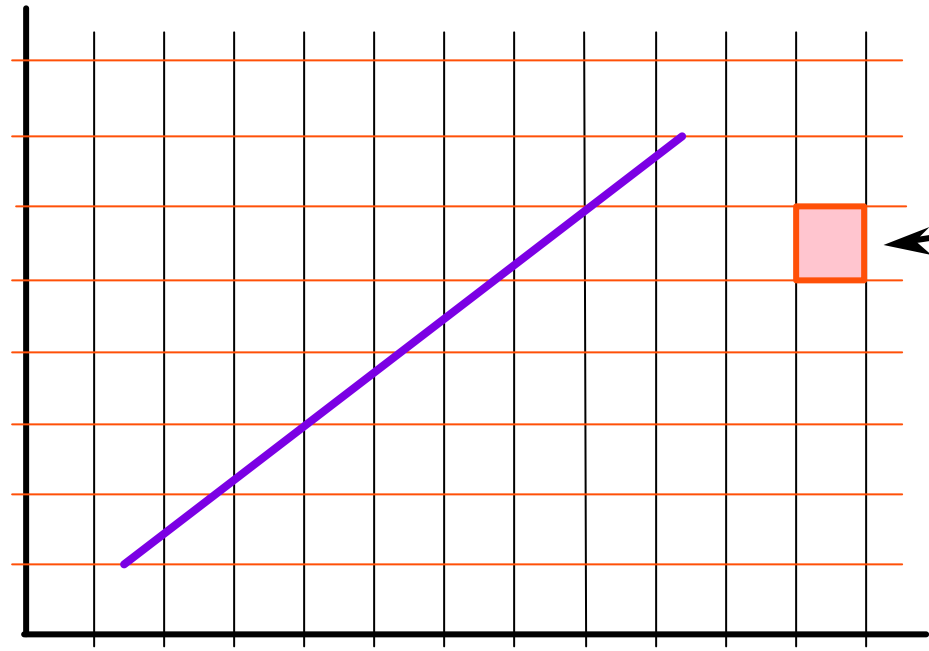
Time-Frequency Mappings



Dyadic Wavelet Transform

Frequency

Time



$$\Delta f \geq \frac{1}{2\Delta t}$$

(Gabor logon)

Gabor Transform

(Windowed STFT)

References
Strang & Nguyen
Qian & Chen

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Gabor versus Wavelet Analysis

- A Gabor transform engine is well suited to bandpass signals, well removed from video baseband, due to the lack of scale invariance
- Wavelets are well suited to natural phenomena analysis, which often exhibit scale invariance due to fractal or 1/f physical processes. These signals are most often video, or video-like baseband RF.
- Wavelet methods are naturally implemented as a decimation cascade, in iterated fashion, with processing demands of $\mathcal{O}(N)$. This is a primary advantage of the dyadic wavelet expansion.
- Gabor processing uses the FFT to achieve very fine time-frequency resolution, with processing demands of $\mathcal{O}(N \log_2(N) + 4N)$.
- In the time invariant limit, Gabor processing devolves to block overlap add/save methods, which are universally used for long convolutions.

Discrete Gabor Expansion

(Ref: Qian & Chen)

This is a linear transform, with a well known inverse form, so signals may be reconstructed from the Time-Frequency domain:

$$\hat{s}(t) = \sum_{m} g(t - mT) \sum_{n=0}^{N-1} C_{m,n} e^{i2pnt/N}$$

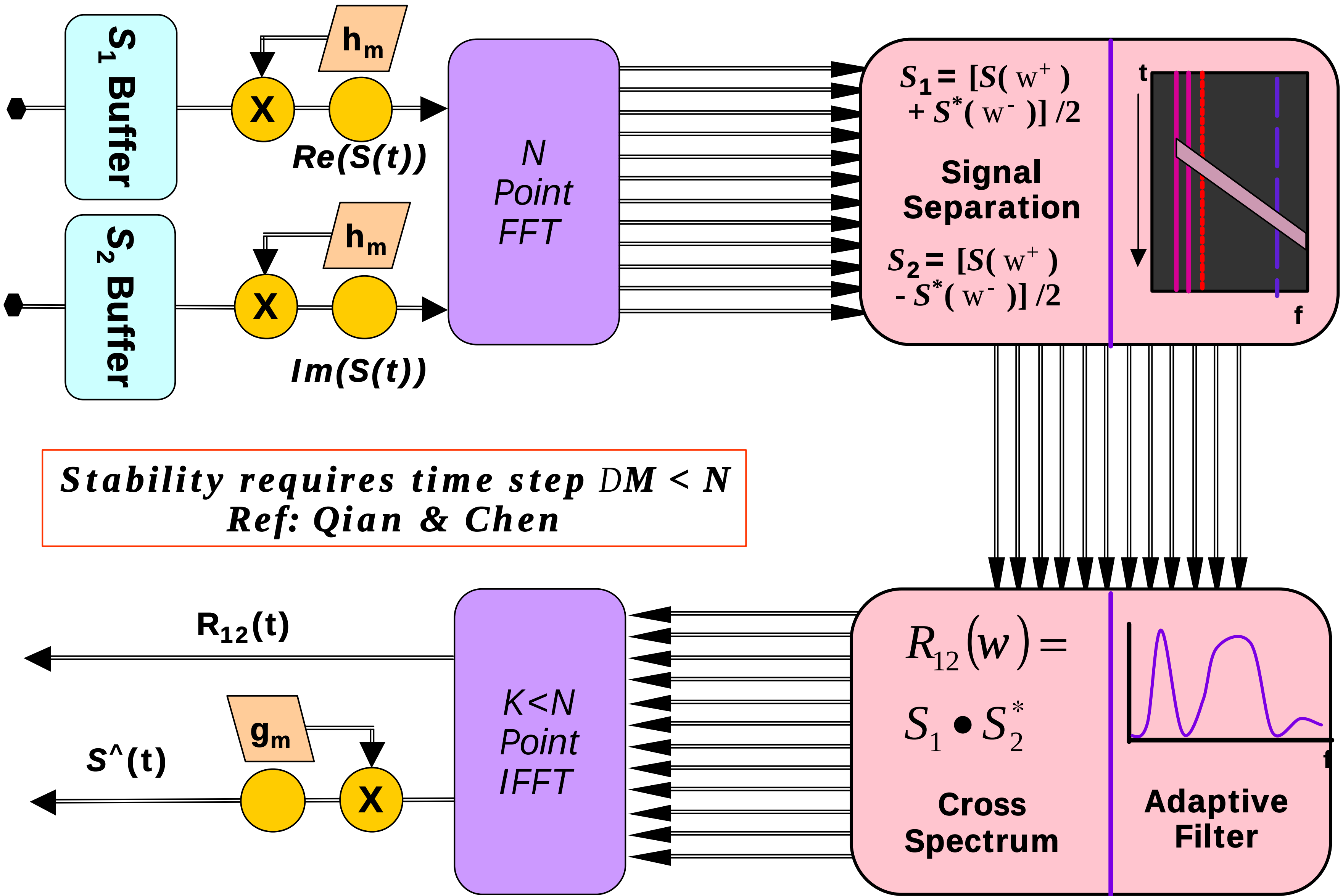
Frequency channel response is controlled by the analysis window h(t), just as in conventional spectral analysis.

$$C_{m,n} = \sum_{t} s(t) h\left(t - mT\right) e^{-i2pnt/N}$$

$$a = N/T \geq 1.0$$

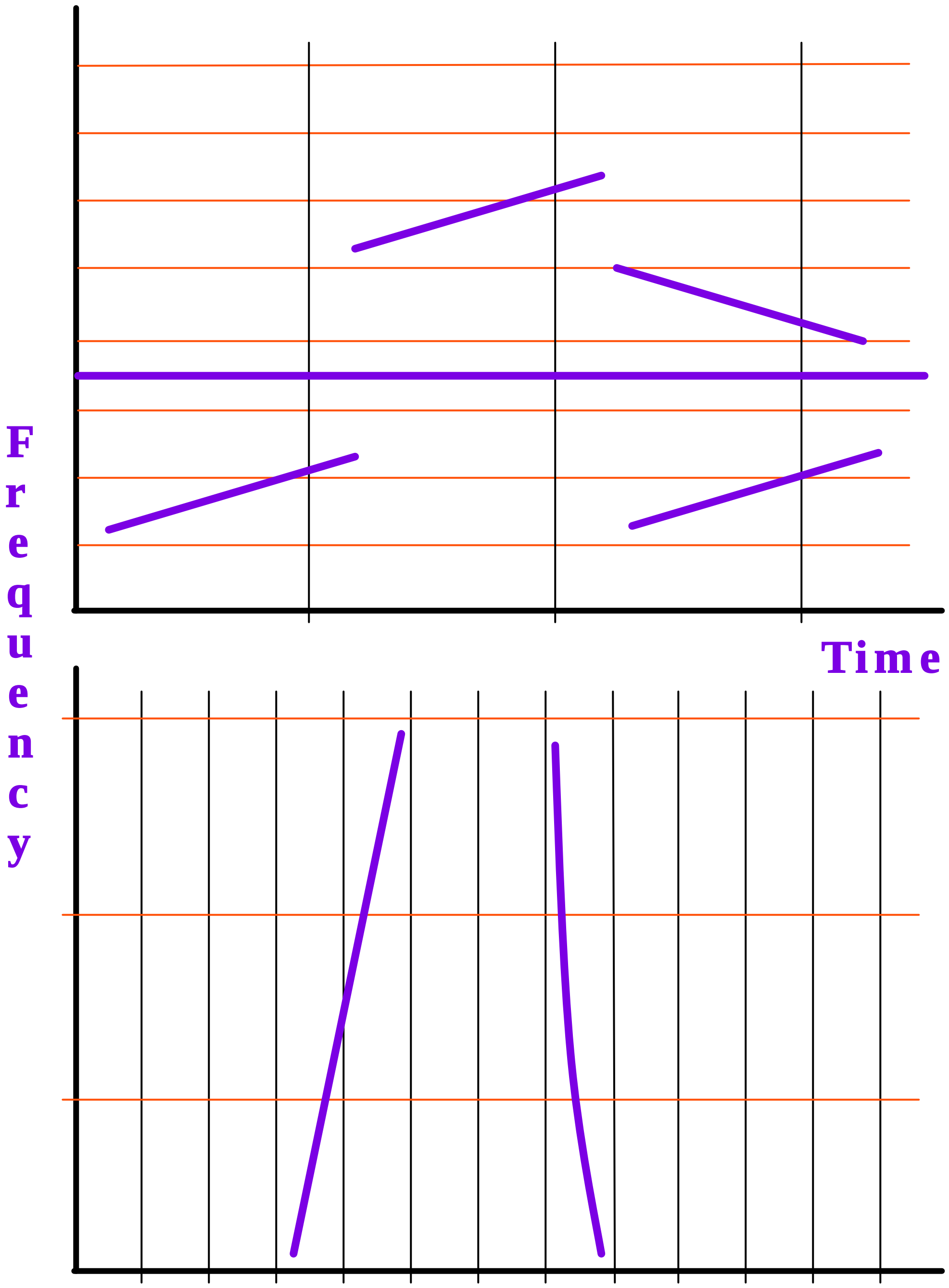
The reconstruction window g(t) is sensitive to h and a . **Many analysts use** $STFT = \left|C_{m,n}\right|$ **to analyze T-F agile data.**

Gabor Filter Bank Processor



Stability requires time step DM < N
Ref: Qian & Chen

Gabor Transforms



Long Time,

Narrowband

Fourier Reciprocity (Uncertainty)

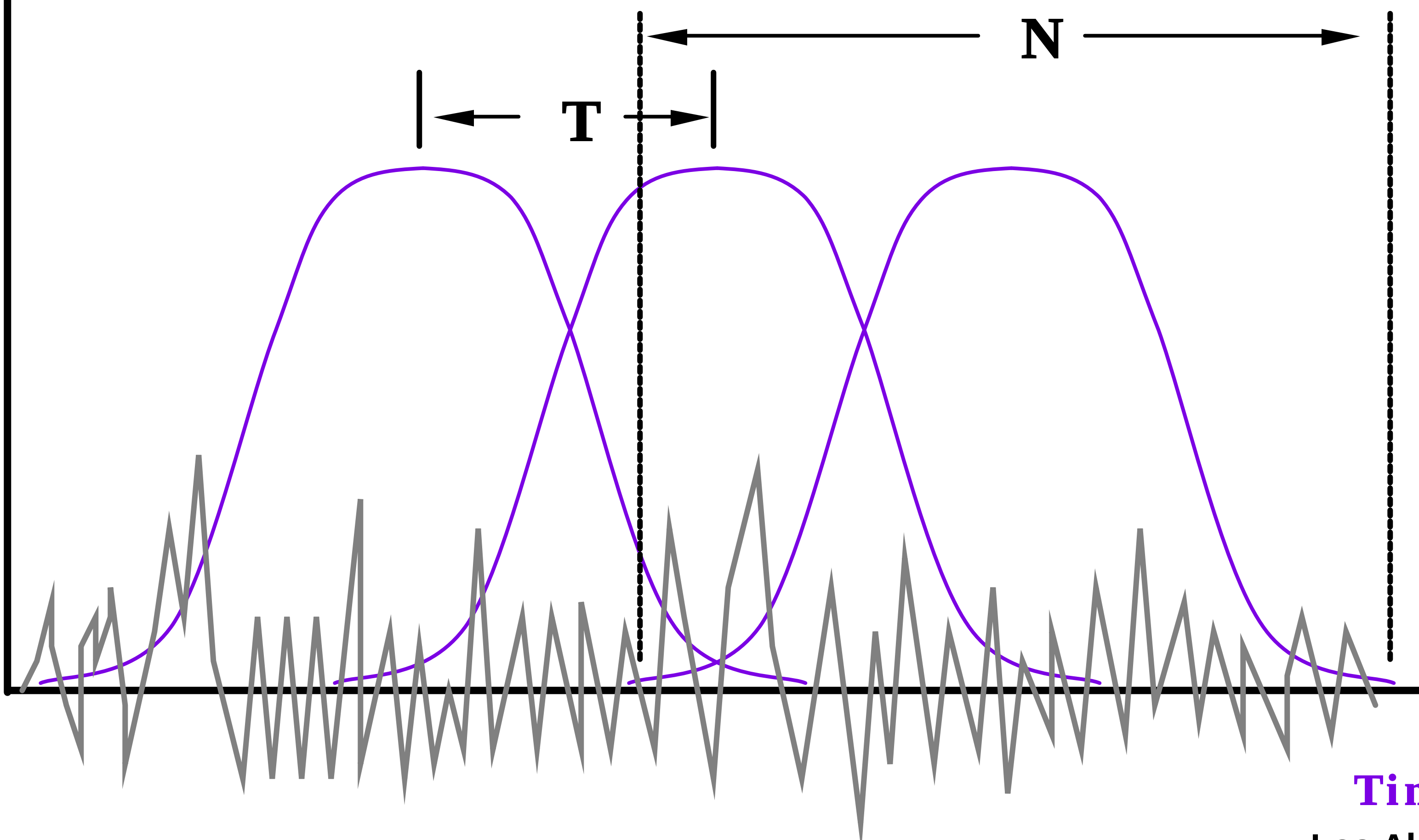
$$\boxed{\Delta f \geq \frac{1}{2\Delta t}}$$

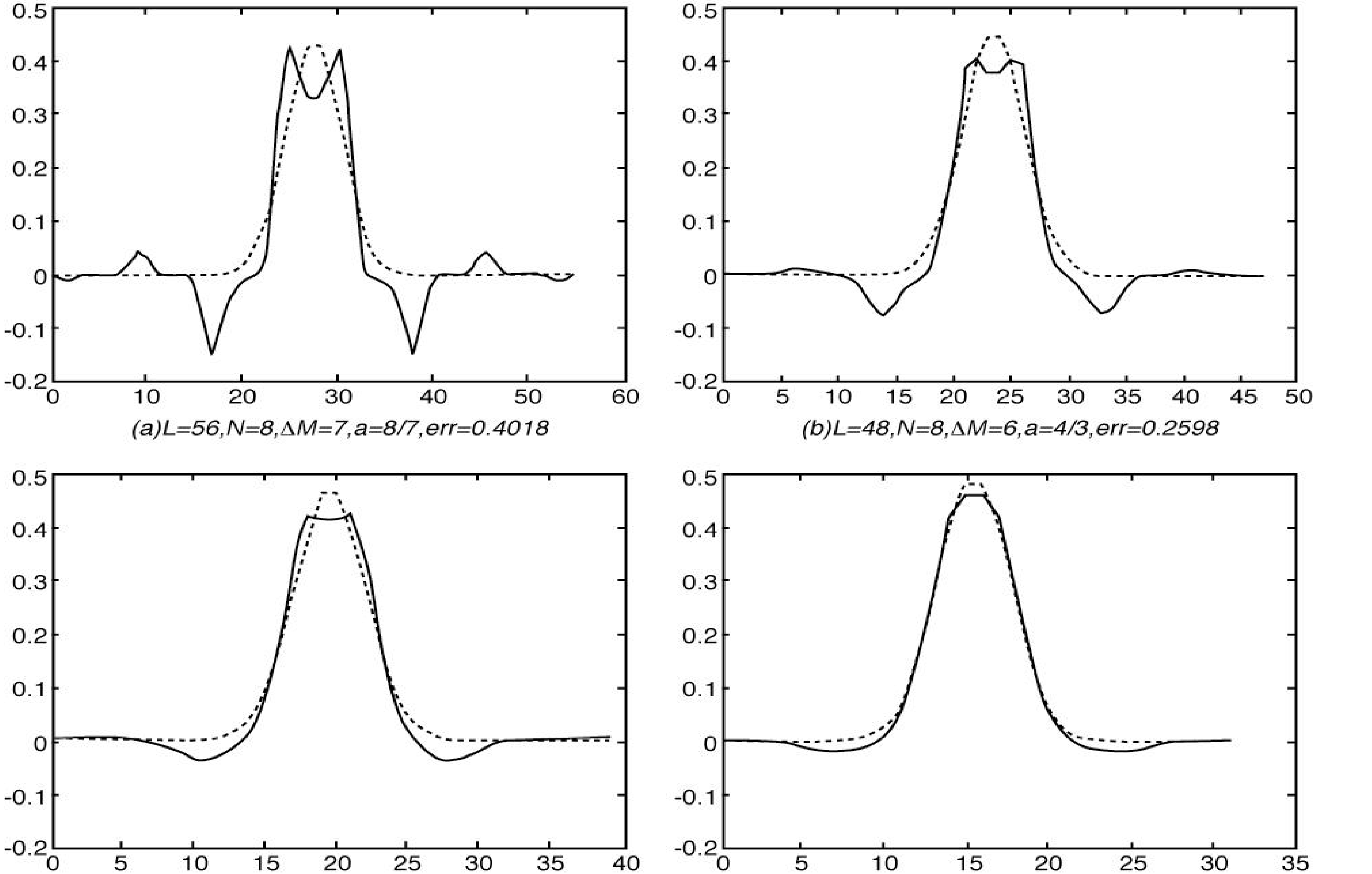
Short Time,

Wideband

Gabor Analysis Windows

$$a = \frac{N}{T} = Oversampling \quad Ratio$$





(a)L=56,N=8,ΔM=7,a=8/7,err=0.4018

(b)L=48,N=8,ΔM=6,a=4/3,err=0.2598

(c)L=40,N=8,ΔM=5,a=8/5,err=0.1628

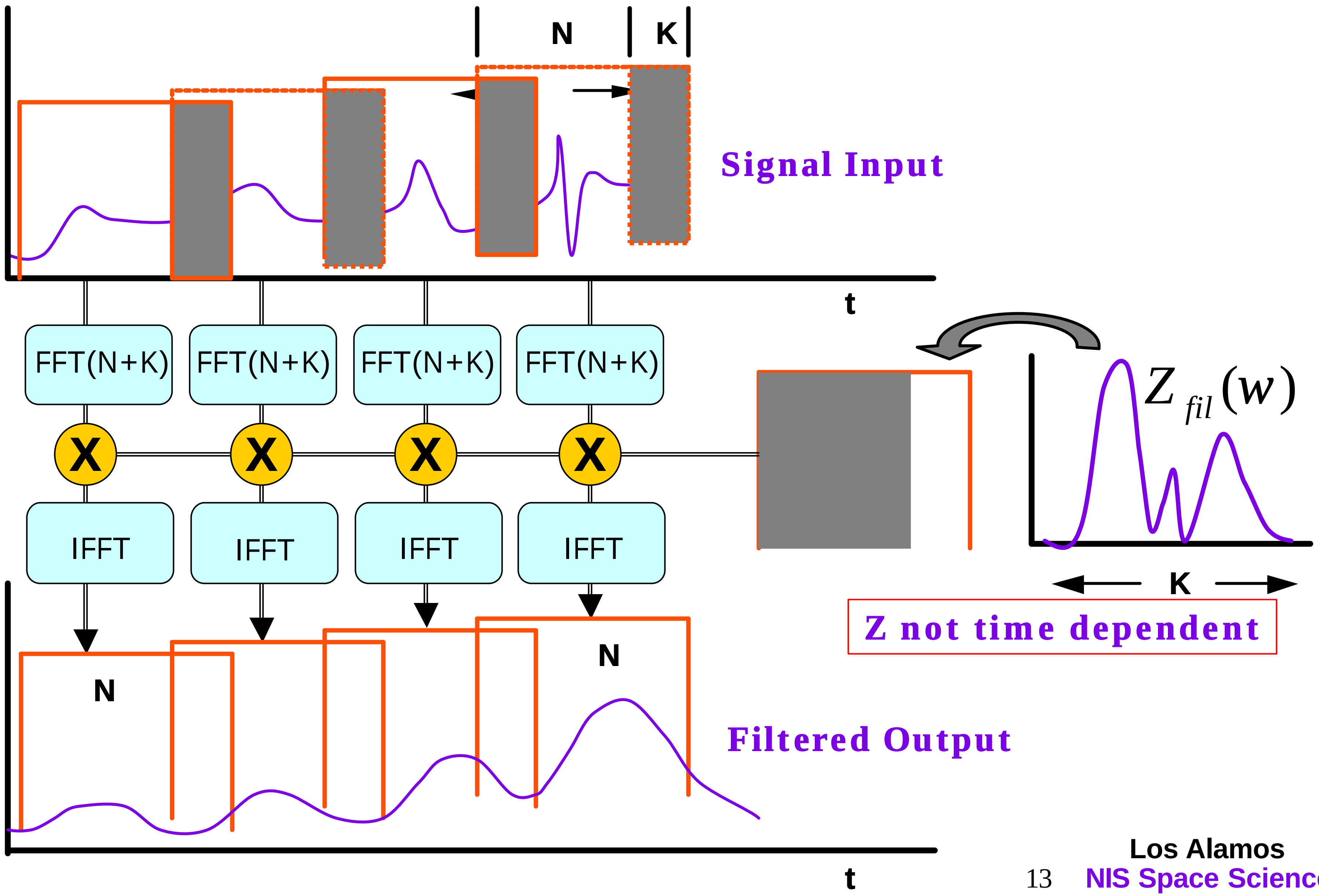
(d)L=32,N=8,ΔM=4,a=2,err=0.0865

(Courtesy: Qian & Chen, **a**= a)

Key Windowing Concepts

- **In the general case, a non-rectangular analysis and synthesis window are required for proper non-aliased operation.**
- **If the windows are shorter than the desired FFT size, and the filtering is time-invariant, then windows can be deleted, as shown next slide**
- **The analysis window function will determine the resolution and leakage of the time-frequency data in the Gabor domain.**
  - **Isolated Channels (e.g.- FDM channel separation)**
  - **slight overlap (e.g.- Multi-channel analysis & reconstruction)**
  - **substantial overlap (e.g.-Magnitude Spectral analysis)**
- **Modern filter designs, & wavelet kernel generation methods, have substantially improved filtering performance of same length windows**
- **Synthesis optimality is achieved by setting the oversampling rate to allow *h(t)* and *g(t)* to be nearly identical, usually with a >2.**

Rectangular Windowing for a Constant Z Vector



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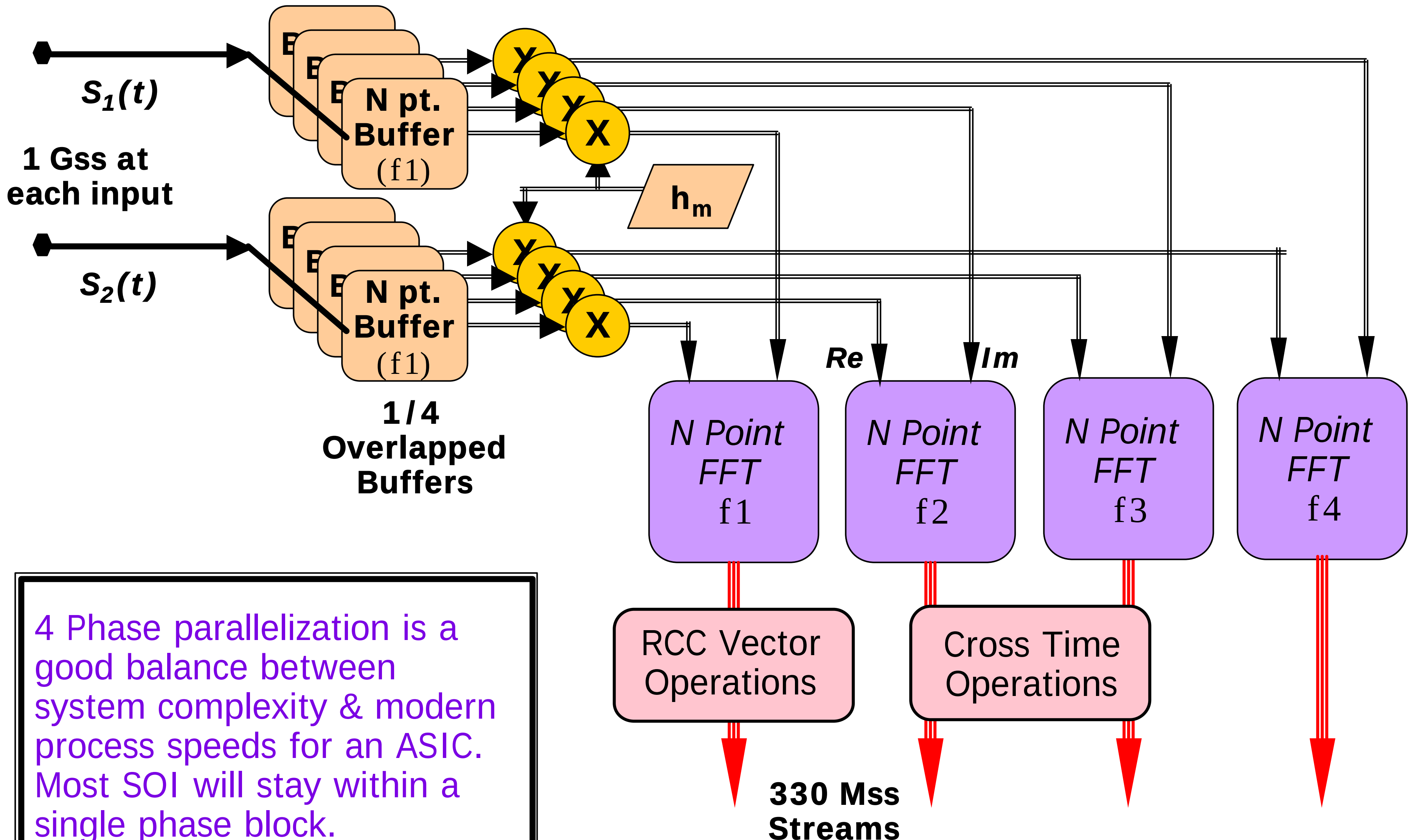
500 MHz Input Processor



4 Phase parallelization is a good balance between system complexity & modern process speeds for an ASIC. Most SOI will stay within a single phase block.

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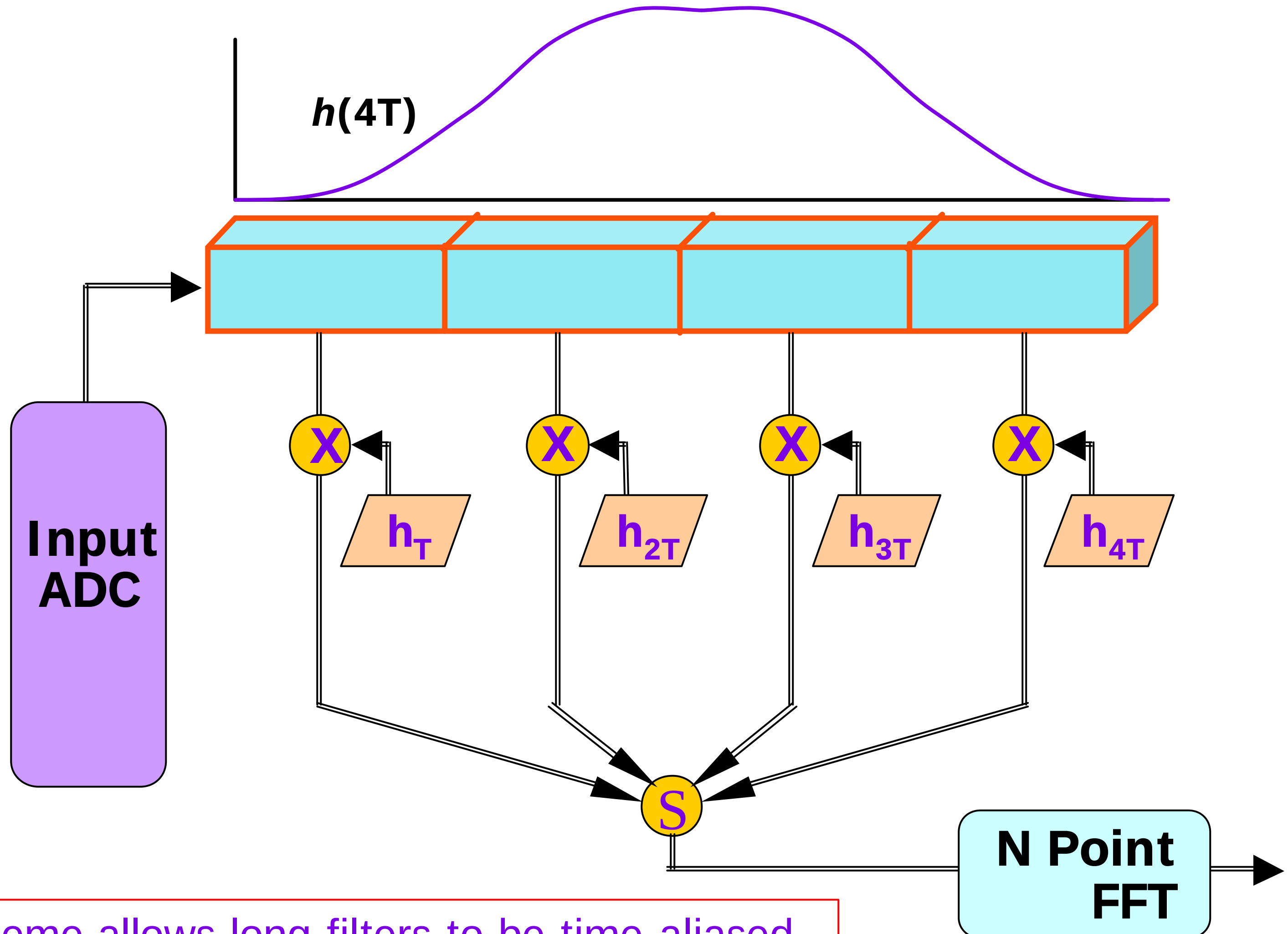
Quad Aliased Time Window



This scheme allows long filters to be time aliased, giving narrow mainlobes with low sidelobes (Crochiere & Rabiner, Section 7.2.5)

Multi-Phase Analysis Window

- ***s(t,t+T)*** **is an input time vector of length m**
- **h(t) is a quadruple length analysis window**
- **T is the block length for time-frequency analysis**
- **This scheme allows long filters to be time aliased, giving narrow mainlobes with low sidelobes (Crochiere & Rabiner, Section 7.2.5)**
- **A four phase layout allows 4X rate reduction in the Z vectors for FFT**

$$\begin{bmatrix} . & . & h_1 & h_2 & h_3 & h_4 & . & . \\ . & . & . & h_1 & h_2 & h_3 & h_4 & . \\ . & . & . & . & h_1 & h_2 & h_3 & h_4 \\ . & . & . & . & . & h_1 & h_2 & h_3 \end{bmatrix} * \begin{bmatrix} . \\ . \\ s(1,T) \\ s(T^+,2T) \\ s(2T^+,3T) \\ s(3T^+,4T) \\ . \\ . \end{bmatrix} = \begin{bmatrix} z_{f1}(t) \\ z_{f2}(t) \\ z_{f3}(t) \\ z_{f4}(t) \end{bmatrix}$$

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Analytic Signal Relations

The analytic signal can be derived via two routes:

a. Causality-- s(t)=0 for t<0

b. Analyticity-- z(t) satisfies the Cauchy-Riemann conditions

Either route requires Im(z) to be the Hilbert Transform of Re(z):

$$z(t) = s(t) + i\,\hat{s}(t) \quad where \quad \hat{s}(t) = ?\,\big(s(t)\big) = \frac{1}{p}\int_0^{\infty} \frac{s(t)}{t - t}\, dt$$

In the frequency domain, these relations become very simple:

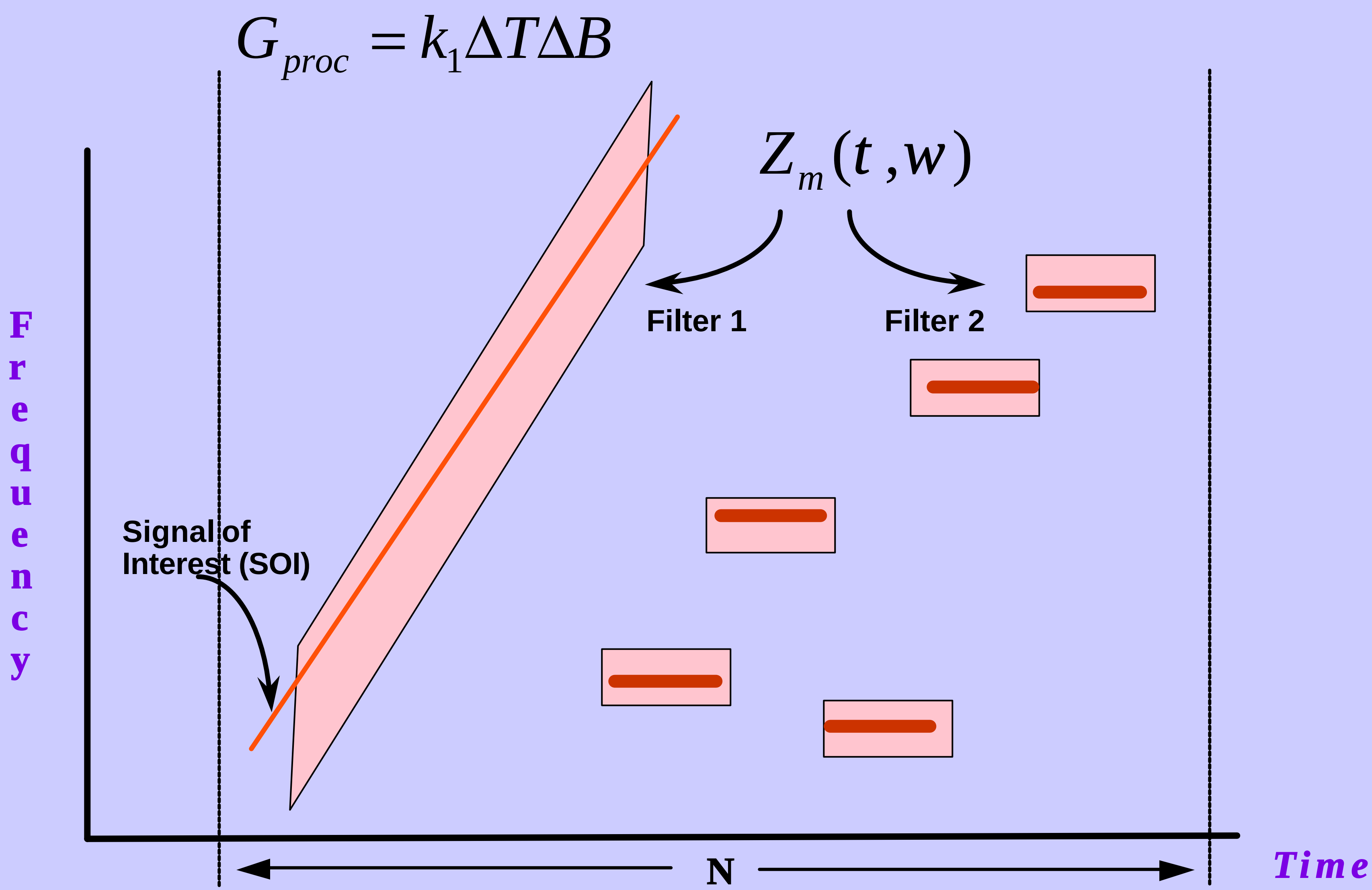
$$\begin{aligned} Z(w) &= S(w) \;\; for \; w = 0, w_N \\ &= 2 \bullet S(w) \;\; for \; w > 0 \;\; and \; w \neq w_N \\ &= 0 \;\; for \; w < 0 \end{aligned}$$

z(t) allows the magnitude and phase of s(t) to be manipulated

Blank Slide for Analytic Signal Example

- **Cannot export from LV3.1**

Single Time-Block Gabor Matched Filters



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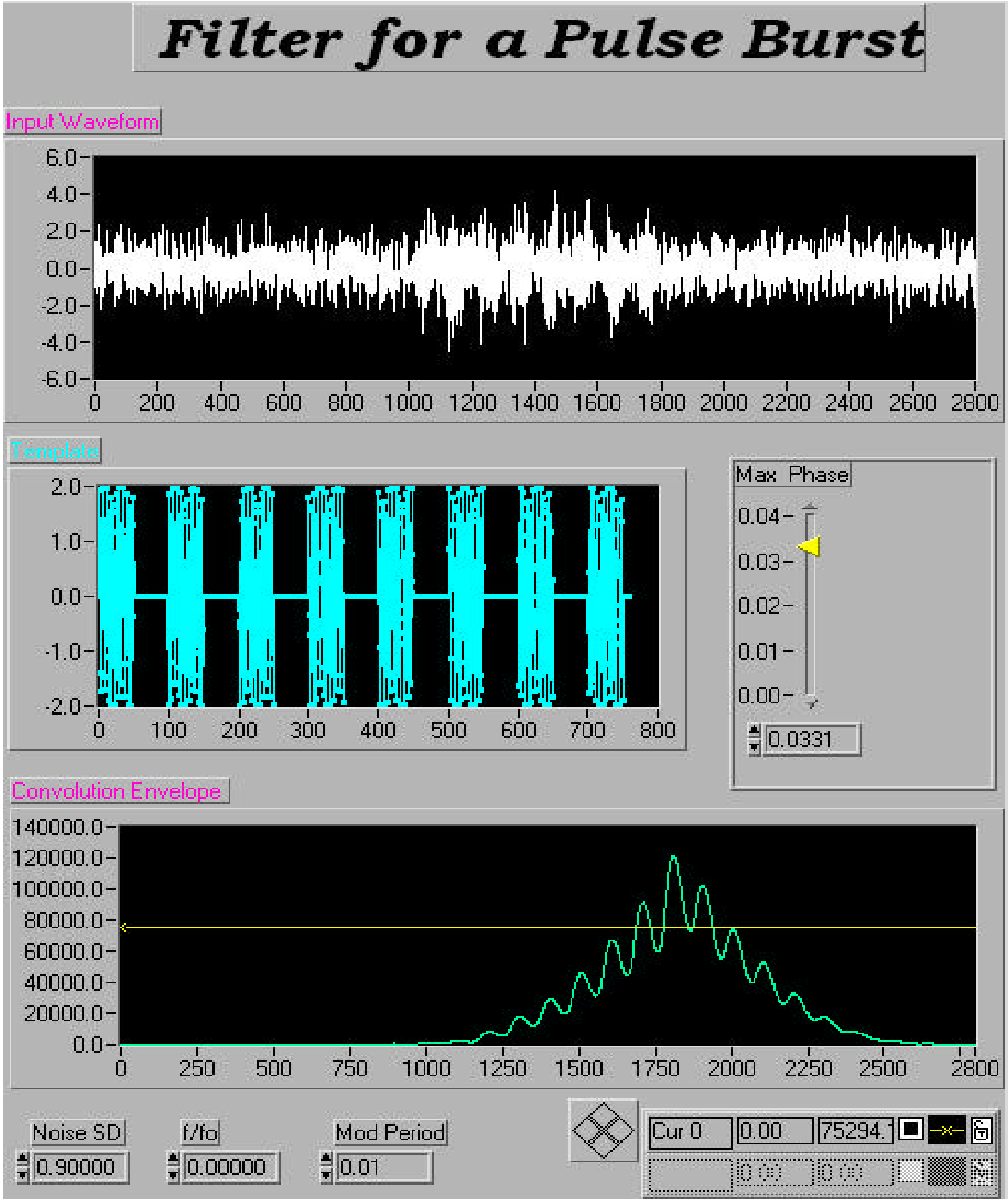

LabVIEW simulation of T-F matched filter

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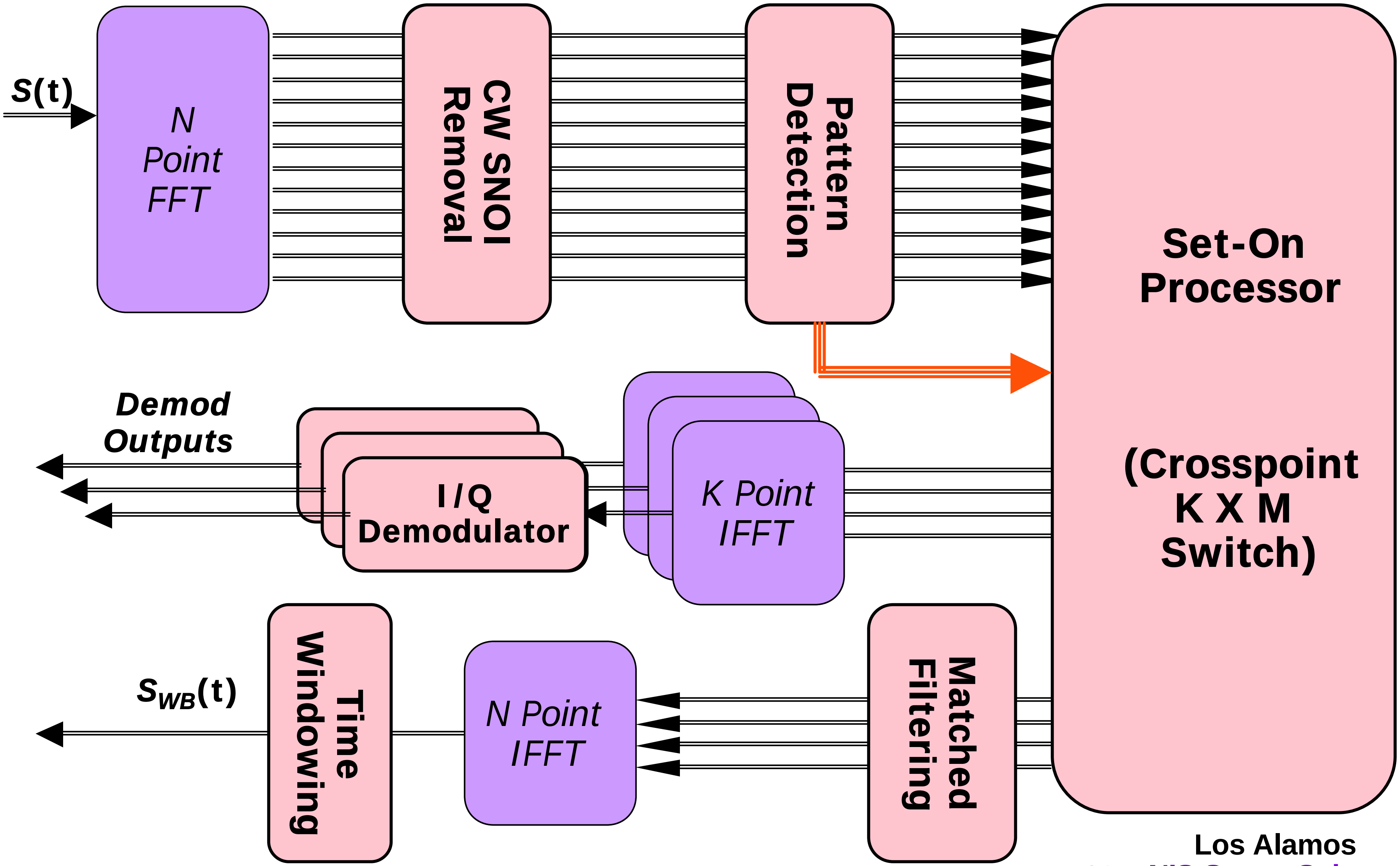
Gabor Domain Processing

SOI Recognition & Digital Set-On Receivers



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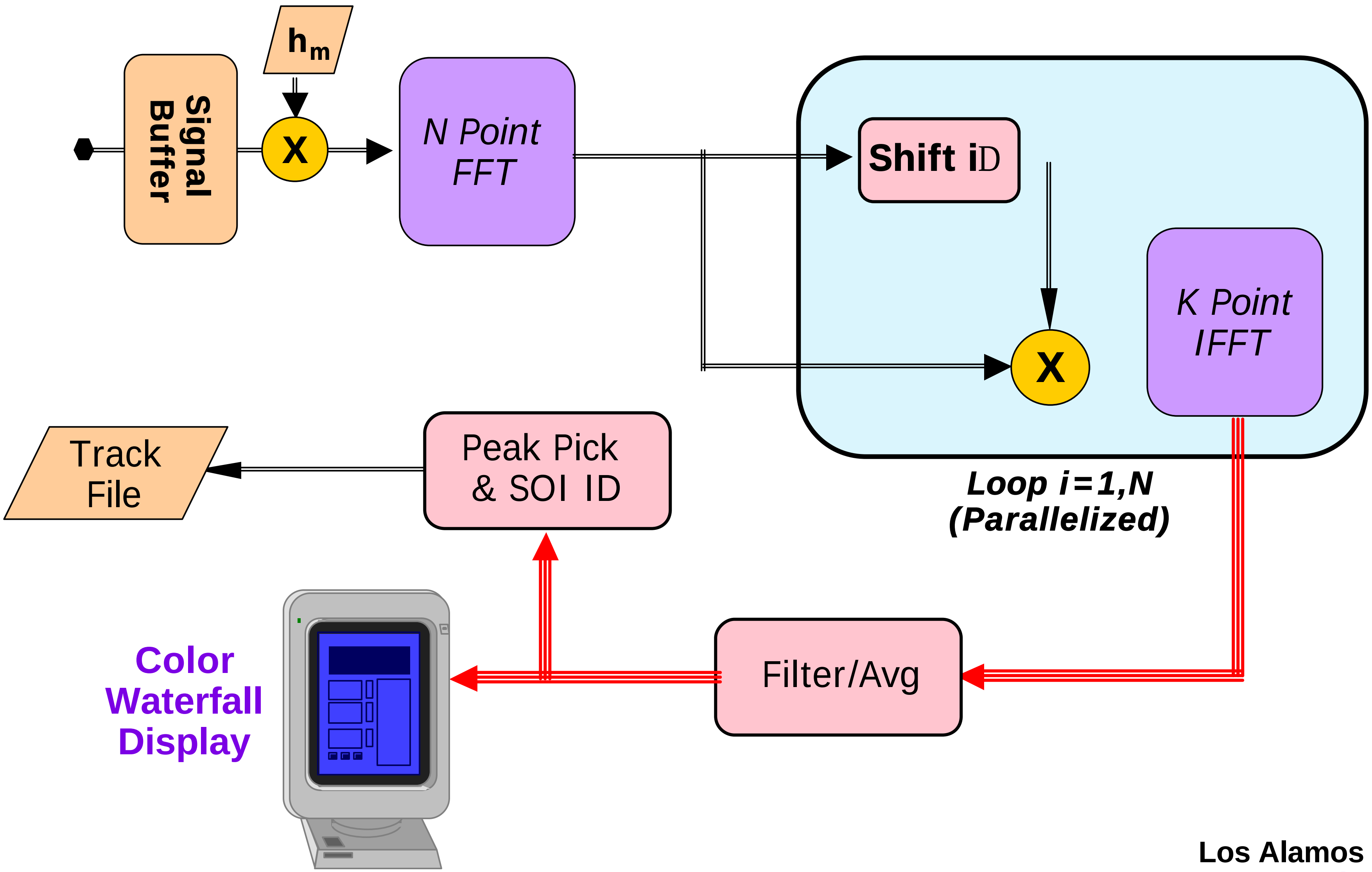
Gabor Domain Processing

Cyclo-Stationary Search via Wigner-Ville Analysis



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Key Wideband DSP Processor Points

- A Gabor processor allows all known analog functions performed in radio receivers to be implemented as programmable code, which can be updated as new techniques come, and/or the environment changes
- Since the Gabor processor is an explicit time-frequency analysis engine, signals which are agile can be attacked coherently in their home domain.
- Pattern recognition and other feature identification techniques can be applied directly to the internal time-frequency datastream, where signal separation has already occurred in the Gabor transform.
- Modern ASIC and FPGA capabilities allow an entire cellular baseband or HDTV video channel to be manipulated and processed in one stream.
- One forward transform engine can feed thousands of inverse transform engines, so that sub-band processing for each active signal in band is quite feasible in a multi-rack system

Conclusions

- We have discussed the combination of mathematical principles and algorithmic elements that form a working detection & estimation processor for RF and video.
- As seen in the references, this framework has been constructed over the last 50 years, and some portions of the theory are widely used in the Radar community for matched filter detection and return processing.
- New hardware from Catalina and other FFT ASIC developers, combined with Reconfigurable FPGA technology from a major DARPA initiative, make a fully flexible Gabor processor feasible now at 100 Mss, and up to 1200 Mss with relatively straightforward parallelization.
- Crossover to large commercial markets in video and communications processing is driving a new partnership with the Motorola PowerPC team, and can help reduce development costs for the Government

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(in time order)

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